The Review Problems on the next 3 pages

ARE NOT COMPREHENSIVE

REVIEW FOR THE FINAL.

However, they do give several of the two types of word problems most commonly confused by students, namely

- Related Rates
- Optimization

The Review Problems on the last two pages are also not comprehensive final exam review.

However, they cover topics which are often difficult for students, namely:

- Volumes
- $\ln(x)$
- e^x

CALCULUS WORD PROBLEMS

	Related Rates	Optimization
variables	usually several variables always includes t unless specified, all change w/ time	if more than two, use secondary to subst for one.
derivative	always WRT t = time chain rule $\frac{d(var)}{dt}$ for every variable	WRT independent variable
solve for	rate requested	variable requested.
method	differentiate, subst, isolate.	differentiate find CV 1st or 2nd deriv test to confirm max or min.
special notes:	no secondary eq'n	primary = function to differentiate (calc) secondary = eq'n with value, solve + subst (algebra)
	time is always essential	time is not usually included
units:	__ per min (or sec or hour...)	as given in question

Math 250 Review Problems

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1) Find the absolute extrema of the function on the closed interval
   a. $f(x) = \dfrac{2x}{x^2+1}$, $[-2,2]$
   b. $h(t) = \tan\left(\dfrac{\pi t}{8}\right)$, $[0,2]$

2) Differentiate
   a. $g(x) = \ln(x^x)$
   b. $f(x) = (\ln x)^x$
   c. $k(x) = \sqrt{7}^{\sqrt{7x}}$
   d. $h(x) = \sqrt{7}^{\sqrt{7x}}$

Solve the problem.

3) Water is falling on a surface, wetting a circular area that is expanding at a rate of $9\ \mathrm{mm^2/s}$. How fast is the radius of the wetted area expanding when the radius is 131 mm? (Round your answer to four decimal places.)

4) A piece of land is shaped like a right triangle. Two people start at the right angle of the triangle at the same time, and walk at the same speed along different legs of the triangle. If the area formed by the positions of the two people and their starting point (the right angle) is changing at $3\ \mathrm{m^2/s}$, then how fast are the people moving when they are 3 m from the right angle? (Round your answer to two decimal places.)

5) One airplane is approaching an airport from the north at 157 km/hr. A second airplane approaches from the east at 299 km/hr. Find the rate at which the distance between the planes changes when the southbound plane is 27 km away from the airport and the westbound plane is 23 km from the airport. Round to the nearest whole number.

6) Water is being drained from a container which has the shape of an inverted right circular cone. The container has a radius of 7.00 inches at the top and a height of 9.00 inches. At the instant when the water in the container is 8.00 inches deep, the surface level is falling at a rate of 1.4 in./sec. Find the rate at which water is being drained from the container. Give an exact answer and one rounded to the nearest whole number.

7) A man 6 ft tall walks at a rate of 3 ft/sec away from a lamppost that is 23 ft high. At what rate is the length of his shadow changing when he is 60 ft away from the lamppost? (Do not round your answer)

8) Electrical systems are governed by Ohm's law, which states that V = IR, where V = voltage, I = current, and R = resistance. If the current in an electrical system is decreasing at a rate of 6 A/s while the voltage remains constant at 12 V, at what rate is the resistance increasing (in Ω/sec) when the current is 60 A? (Do not round your answer.)

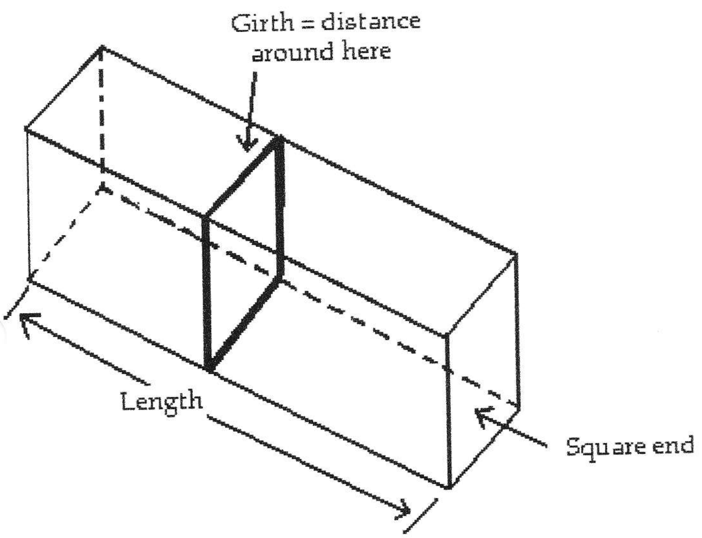
9) The radius of a right circular cylinder is increasing at the rate of 5 in./sec, while the height is decreasing at the rate of 2 in./sec. At what rate is the volume of the cylinder changing when the radius is 13 in. and the height is 8 in.?

10) A rectangular field is to be enclosed on four sides with a fence. Fencing costs $5 per foot for two opposite sides, and $2 per foot for the other two sides. Find the dimensions of the field of area 790 ft^2 that would be the cheapest to enclose. Give exact answers, then rounded to the nearest tenth.

11) From a thin piece of cardboard 20 in. by 20 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume? Give an exact answer, then one rounded to the nearest tenth.

12) A company is constructing an open-top, square-based, rectangular metal tank that will have a volume of 56.5 ft^3. What dimensions yield the minimum surface area? Round to the nearest tenth, if necessary.

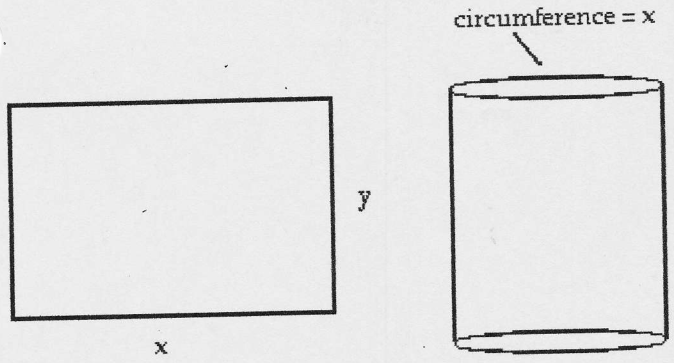
13) A private shipping company will accept a box for domestic shipment only if the sum of its length and girth (distance around) does not exceed 120 in. What dimensions will give a box with a square end the largest possible volume?



14) A long strip of sheet metal 12 inches wide is to be made into a small trough by turning up two sides at right angles to the base. If the trough is to have maximum capacity, how many inches should be turned up on each side?

15) Determine the dimensions of the rectangle of largest area that can be inscribed in a semicircle of radius 3.

16) A rectangular sheet of perimeter 39 cm and dimensions x cm by y cm is to be rolled into a cylinder as shown in part (a) of the figure. What values of x and y give the largest volume?

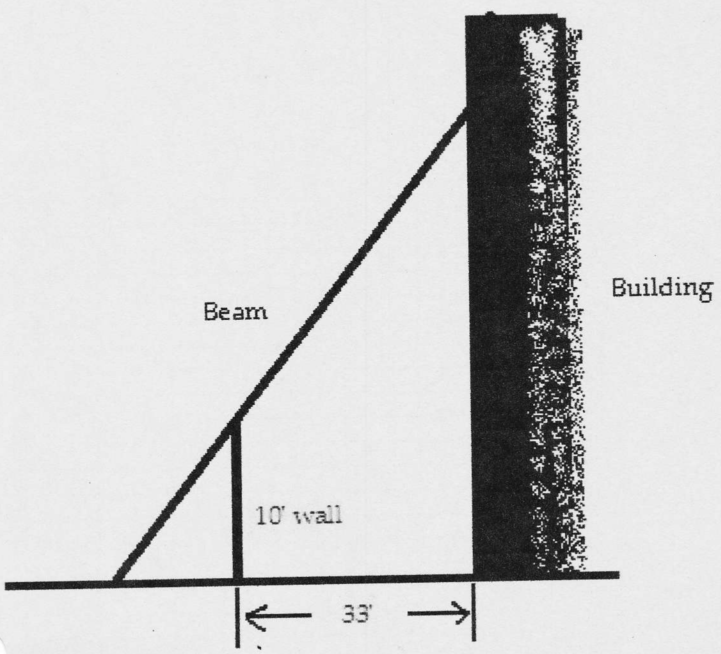


Challenge problems:

17) The volume of a sphere is increasing at a rate of 5 cm^3/sec. Find the rate of change of its surface area when its volume is $\frac{256\pi}{3}$ cm^3. (Do not round your answer.)

18) The volume of a rectangular box with a square base remains constant at 600 cm^3 as the area of the base increases at a rate of 8 cm^2/sec. Find the rate at which the height of the box is decreasing when each side of the base is 18 cm long. (Do not round your answer.)

19) The 10 ft wall shown here stands 33 feet from the building. Find the length of the shortest straight beam that will reach to the side of the building from the ground outside the wall. Round to the nearest tenth.



ANSWERS

1) a. max 1 at x=1, min -1 at x=-1

b. max 1 at x=2, min 0 at x=0

2) a. $g'(x) = \ln(x) + 1$

b. $f'(x) = (\ln x)^{x-1}[1 + \ln(\ln x) \cdot \ln(x)]$

c. $k'(x) = \ln\sqrt{7}\left(\sqrt{7}\right)^{\sqrt{7x+1}}$

d. $h'(x) = \dfrac{\left(\sqrt{7}\right)^{\sqrt{7x+1}}}{2\sqrt{x}}\left(1 + \ln\sqrt{7x}\right)$

3) 0.0109 mm/s

4) $\dfrac{9}{266\pi} mm/\sec \approx 0.0108 mm/\sec$

5) -313 km/hr

6) $\dfrac{-21952\pi}{405} in^3/\sec$, 170 in.^{3}/s

7) $\dfrac{18}{17}$ ft/sec

8) $\dfrac{1}{50}$ Ω/sec

9) 702π in.^{3}/sec

10) $5\sqrt{79} ft$ @\$5, $2\sqrt{79} ft$ @\$2;
17.8 ft @ \$5 by 44.4 ft @ \$2

11) $\dfrac{40}{3} in \times \dfrac{40}{3} in \times \dfrac{10}{3} in$, $V = \dfrac{16000}{27} in^3$;
~13.3 in. × ~13.3 in. × ~3.3 in.;
V=~592.6 in^3

12) 4.8 ft × 4.8 ft × 2.4 ft

13) 20 in. × 20 in. × 40 in.

14) 3 in.

15) $\sqrt{6}$ in x $\dfrac{\sqrt{6}}{2}$ in

16) x = 13 cm; y = $\dfrac{13}{2}$ cm

17) $\dfrac{5}{2}$ cm^2/sec

18) $-\dfrac{400}{27}$ cm/sec

19) 57.7 ft

M250 FE Review Problems (Selected)

① Absolute extrema on interval

a. $f(x) = \dfrac{2x}{x^2+1}$ on $[-2,2]$

C.V.s: $f'(x) = \dfrac{(x^2+1)\cdot 2 - 2x(2x)}{(x^2+1)^2}$ quotient

$= \dfrac{2x^2+2-4x^2}{(x^2+1)^2}$

$= \dfrac{2-2x^2}{(x^2+1)^2}$

$2-2x^2=0$

$2=2x^2$

$1=x^2$

$x=\pm 1$ both in $[-2,2]$

$x^2+1=0$ imaginary

x	$f(x)$
2	0.8
-2	-0.8
1	1
-1	-1

$\boxed{\begin{array}{l} \text{max } 1 \text{ at } x=1 \quad (1,1) \\ \text{min } -1 \text{ at } x=-1 \quad (-1,-1) \end{array}}$

b. $h(t) = \tan\left(\dfrac{\pi t}{8}\right)$ on $[0,2]$

C.V.s $h'(t) = \sec^2\left(\dfrac{\pi t}{8}\right)\cdot\dfrac{\pi}{8}$

$\sec x \neq 0$ (ever)

$h(t)$ and $h'(t)$ undefined

when $\dfrac{\pi t}{8} = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$, etc.

$2\pi t = 8\pi$

$t=4$ not in interval

t	$h(t)$
0	0
2	1

$\boxed{\begin{array}{l} \text{max } 1 \text{ at } x=2 \quad (2,1) \\ \text{min } 0 \text{ at } x=0 \quad (0,0) \end{array}}$

② Differentiate

a. $g(x) = \ln(x^x)$

$g(x) = x\ln(x)$

$g'(x) = x\cdot\dfrac{1}{x} + 1\cdot\ln x$ product

$\boxed{g'(x) = 1+\ln x}$

b. $f(x) = (\ln x)^x$ no log prop!

$y = (\ln x)^x$ ln differentiation

$\ln y = \ln(\ln x)^x$ take ln

$\ln y = x\cdot\ln(\ln x)$ ln properties

$\dfrac{1}{y}\dfrac{dy}{dx} = x\cdot\dfrac{1}{\ln x}\cdot\dfrac{1}{x} + 1\cdot\ln(\ln x)$ implicit & product

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{\ln x} + \ln(\ln x)$

$\dfrac{dy}{dx} = y\left(\dfrac{1}{\ln x} + \ln(\ln x)\right)$

$\dfrac{dy}{dx} = (\ln x)^x\left(\dfrac{1}{\ln x} + \ln(\ln x)\right)$

$\dfrac{dy}{dx} = \dfrac{(\ln x)^x}{\ln x}\left(1 + \ln(\ln x)\cdot\ln x\right)$

$\boxed{\dfrac{dy}{dx} = (\ln x)^{x-1}\left[1+\ln(\ln x)\cdot\ln x\right]}$

c. $k(x) = \sqrt{7}^{\sqrt{7}x}$

$k'(x) = \ln\sqrt{7}\cdot\sqrt{7}^{\sqrt{7}x}\cdot\sqrt{7}$ chain rule

$\boxed{k'(x) = \ln\sqrt{7}\cdot\sqrt{7}^{\sqrt{7}x+1}}$

d. $h(x) = \sqrt{7}^{\sqrt{7x}} = \sqrt{7}^{\sqrt{7}\cdot\sqrt{x}}$

$h'(x) = \ln\sqrt{7}\cdot\sqrt{7}^{\sqrt{7}\cdot\sqrt{x}}\cdot\sqrt{7}\cdot\dfrac{1}{2}x^{-1/2}$ chain rule

$\boxed{h'(x) = \dfrac{\ln\sqrt{7}\ \sqrt{7}^{\sqrt{7}\sqrt{x}+1}}{2\sqrt{x}}}$

③ circle area $A = \pi r^2$

$\frac{dA}{dt} = +9 \frac{mm^2}{sec}$

find $\frac{dr}{dt}$ when $r = 133$ mm

4 decimal places

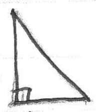
$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ diff WRT time

$9 = 2\pi(133)\frac{dr}{dt}$

$\frac{9}{266\pi} = \frac{dr}{dt}$

$\boxed{\frac{dr}{dt} \approx 0.0108 \frac{mm}{sec}}$

④ right triangle



$\frac{dy}{dt}$ ↑ →

$\frac{dx}{dt}$

$\frac{dx}{dt} = \frac{dy}{dt}$ "same speed"

$\frac{dA}{dt} = 3 \frac{m^2}{sec}$ $x = 3m$

$A = \frac{1}{2}x \cdot y = \frac{1}{2}x^2$

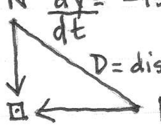
$\frac{dA}{dt} = 2 \cdot \frac{1}{2} \cdot x \cdot \frac{dx}{dt}$

$\frac{dA}{dt} = x \cdot \frac{dx}{dt}$

$3 = 3 \cdot \frac{dx}{dt}$

$\boxed{\frac{1m}{sec} = \frac{dx}{dt}}$

⑤



N $\frac{dy}{dt} = -157 \frac{km}{hr}$

D = distance between

airport @origin

E

$\frac{dx}{dt} = -299 \frac{km}{hr}$

Find $\frac{dD}{dt}$ when $y = 27$ km, $x = 23$ km

round nearest whole number

$D = \sqrt{x^2 + y^2}$

$\frac{dD}{dt} = \frac{1}{2}(x^2+y^2)^{-1/2}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right)$

$\frac{dD}{dt} = \frac{x\frac{dx}{dt} + y \cdot \frac{dy}{dt}}{\sqrt{x^2+y^2}}$

$\frac{dD}{dt} = \frac{(23)(-299) + (27)(-157)}{\sqrt{23^2 + 27^2}}$

$= \frac{-11116}{\sqrt{1258}}$

≈ -313.4

$\frac{dD}{dt} \approx \boxed{-313 \frac{km}{hr}}$

⑥

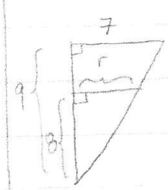
$R = 7$ in

$H = 9$ in

$h = 8$ in

$\frac{dh}{dt} = -1.4 \frac{in}{sec}$ ← have h, dont want r.

find $\frac{dV}{dt}$ exact & nearest whole.



similar Δs $\frac{9}{8} = \frac{7}{r}$

$9r = 56$

$r = \frac{56}{9} = 6\frac{2}{9}$ in

But more importantly: $\frac{h}{r} = \frac{9}{7}$ for all h & r

$r = \frac{7}{9}h$

Subst into $V = \frac{1}{3}\pi r^2 h$ to remove r.

$V = \frac{1}{3}\pi\left(\frac{7}{9}h\right)^2 h$

$V = \frac{1}{3}\pi\left(\frac{49}{81}h^2\right)h$

(6) cont

$V = \frac{49}{243}\pi h^3$

$\frac{dV}{dt} = \frac{49}{243}\pi \cdot 3h^2 \frac{dh}{dt}$

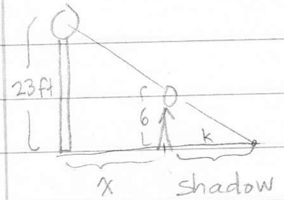
$\frac{dV}{dt} = \frac{49\pi}{81} h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{49\pi}{81} (8)^2 \cdot (-1.4)$

$\frac{dV}{dt} = \boxed{\frac{-21952\pi}{405} \frac{in^3}{sec}}$

$\approx \boxed{170 \frac{in^3}{sec}}$

(7)



$\frac{dx}{dt} = \frac{3 ft}{sec}$

find $\frac{dk}{dt}$ when $x = 60$.

similar triangles : want k

$\frac{k}{6} = \frac{k+x}{23}$

$23k = 6k + 6x$

$17k = 6x$

$17\frac{dk}{dt} = 6\frac{dx}{dt}$

$17\frac{dk}{dt} = 6 \cdot 3$

$\boxed{\frac{dk}{dt} = \frac{18}{17} \frac{ft}{sec}}$

(8) $V = IR$ V=voltage $\frac{dV}{dt} = 0$, $V = 12$

I=current $\frac{dI}{dt} = -6 \, A/sec$

R=resistance   find $\frac{dR}{dt}$

when $I = 60A$.

$\frac{dV}{dt} = I \cdot \frac{dR}{dt} + R \cdot \frac{dI}{dt}$ product rule

$0 = 60 \cdot \frac{dR}{dt} + \boxed{.2} \cdot (-6)$

Find R: $V = IR$

$12 = 60 \cdot R$

$0.2 = \frac{1}{5} = \frac{12}{60} = R$

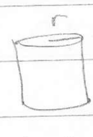
$0 = 60\frac{dR}{dt} - 1.2$

$1.2 = 60\frac{dR}{dt}$

$\frac{1.2}{60} = \frac{dR}{dt}$

$\frac{dR}{dt} = \boxed{\frac{1\,\Omega}{50\ sec}} = \boxed{0.02 \frac{\Omega}{sec}}$

(9)



$\frac{dr}{dt} = 5 \frac{in}{sec}$ $\frac{dh}{dt} = -2 \frac{in}{sec}$

Find $\frac{dV}{dt}$ when $r = 13$ in and $h = 8$ in

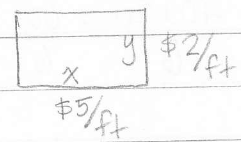
$V = \pi r^2 h$

$\frac{dV}{dt} = \pi\left[2r\frac{dr}{dt} \cdot h + r^2 \cdot \frac{dh}{dt}\right]$ product rule

$\frac{dV}{dt} = \pi\left[2 \cdot 13 \cdot 5 \cdot 8 + 13^2 \cdot (-2)\right]$

$= \boxed{702\pi \frac{in^3}{sec}}$

(10)



$A = 790 ft^2$

lowest cost exact, nearest tenth.

primary objective $C = 2(5x) + 2(2y)$

$C = 10x + 4y$

secondary constraint $A = xy = 790$

(10) cont

$y = \frac{790}{x} = 790x^{-1}$

$C = 10x + 4(790x^{-1})$

$C = 10x + 3160x^{-1}$

$C'(x) = 10 - 3160x^{-2} = 0$

$10 = \frac{3160}{x^2}$

$10x^2 = 3160$

$x^2 = 316$

$x = \pm\sqrt{316}$

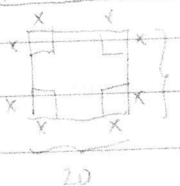
c': $+$ $\nearrow$ $-\sqrt{316}$ $-$ $\searrow$ $+\sqrt{316}$ $+$ $\nearrow$

min @ $x = \sqrt{316} = 2\sqrt{79}$

$y = \frac{790}{2\sqrt{79}} = \frac{10\sqrt{79}\sqrt{79}}{2\sqrt{79}} = 5\sqrt{79}$

$\boxed{x = 2\sqrt{79} \text{ ft @ \$5/ft}} \approx 17.8$ ft

$\boxed{y = 5\sqrt{79} \text{ ft @ \$2/ft}} \approx 44.4$ ft

(11) 
20 in max volume, dimensions exact, tenth

$V = (20-2x)^2 \cdot x = x(400 - 80x + 4x^2)$

$V = 400x - 80x^2 + 4x^3$

$V'(x) = 400 - 160x + 12x^2 = 0$

$12x^2 - 160x + 400 = 0$

$3x^2 - 40x + 100 = 0$

$b^2 - 4ac = (-40)^2 - 4(3)(100) = 400$

$x = \frac{40 \pm \sqrt{400}}{2(3)} = \frac{40 \pm 20}{6} = \frac{60}{6}, \frac{20}{6}$

$x = 10, \frac{10}{3}$

$V'(x)$: $+$ $\nearrow$ $\frac{10}{3}$ $-$ $\searrow$ 10 $+$ $\nearrow$

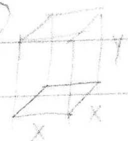
max @ $x = \frac{10}{3}$ in

$20 - 2\left(\frac{10}{3}\right) = 20 - \frac{20}{3} = \frac{40}{3}$ in

$\boxed{\text{dimensions } \frac{10}{3} \text{ in} \times \frac{40}{3} \text{ in} \times \frac{40}{3} \text{ in}}$ exact

$\boxed{3.3 \text{ in} \times 13.3 \text{ in} \times 13.3 \text{ in}}$ nearest tenth

$\boxed{\text{max volume } \frac{16000}{27} \text{ in}^3} \approx \boxed{592.6 \text{ in}^3}$

(12) 
$V = 56.5$ in^3

min surface area, no top.

tenth. dimensions

$V = x^2 y = 56.5$ $\qquad y = \frac{56.5}{x^2}$

$A = x^2 + 4xy$

$A = x^2 + 4x\left(\frac{56.5}{x^2}\right)$

$A = x^2 + 226x^{-1}$

$A'(x) = 2x - 226x^{-2} = 0$

$2x = \frac{226}{x^2}$

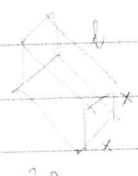
$x^3 = 113$

$x = \sqrt[3]{113} \approx 4.8$ in

$y = \frac{56.5}{(\sqrt[3]{113})^2} \approx 2.4$ in

$A'(x)$: $-$ $\searrow$ $\sqrt[3]{113}$ $+$ $\nearrow$ min.

dimensions $\boxed{4.8 \text{ in} \times 4.8 \text{ in} \times 2.4 \text{ in}}$

(13) 
girth $= 4x$

accept: $l + 4x < 120$ in

max volume, find dimensions

$V = x^2 l$

$l + 4x = 120 \rightarrow l = 120 - 4x$

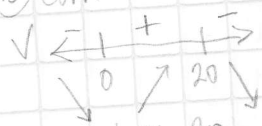
$V = x^2(120 - 4x) = 120x^2 - 4x^3$

$V'(x) = 240x - 12x^2 = 0$

$12x(20 - x) = 0$

$x = 0, 20$

(13) cont



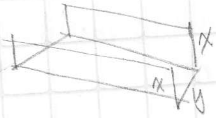
max at $x = 20$

$\ell = 120 - 4(20)$

$= 40$

$\boxed{20\text{ in} \times 20\text{ in} \times 40\text{ in}}$

(14)



max area
find x

$2x + y = 12 \qquad y = 12 - 2x$

cross-sectional area $A = xy$

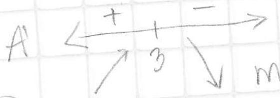
$A = x(12 - 2x)$

$A = 12x - 2x^2$

$A'(x) = 12 - 4x = 0$

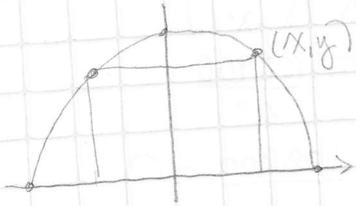
$12 = 4x$

$3 = x$



$\boxed{x = 3\text{ in}}$ each side

(15) Rectangle of largest area inscribed in semicircle radius 3



$x^2 + y^2 = 3$

$y^2 = 3 - x^2$

$y = \sqrt{3 - x^2}$

$A = 2xy$

$A = 2x(3 - x^2)^{1/2}$

$A'(x) = 2\left[x \cdot \frac{1}{2}(3 - x^2)^{-1/2}(-2x) + (3 - x^2)^{1/2}\right]$

$A'(x) = 2\left[-x^2(3 - x^2)^{-1/2} + (3 - x^2)^{1/2}\right]$

$= 2(3 - x^2)^{-1/2}\left[-x^2 + 3 - x^2\right]$

$A'(x) = \dfrac{2(-2x^2 + 3)}{\sqrt{3 - x^2}}$

$A'(x) = 0 \qquad -2x^2 + 3 = 0$

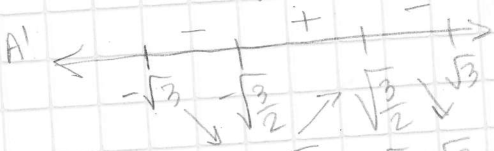
$3 = 2x^2$

$\pm\sqrt{\dfrac{3}{2}} = x$

$A'(x)$ undef $\quad 3 - x^2 = 0$

$3 = x^2$

$\pm\sqrt{3} = x$



max when $x = \sqrt{\dfrac{3}{2}} = \dfrac{\sqrt{3}}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{6}}{2}$

dimensions $\quad 2x = 2\dfrac{\sqrt{6}}{2} = \sqrt{6}$

$y = \sqrt{3 - 3/2} = \sqrt{\dfrac{3}{2}} = \dfrac{\sqrt{6}}{2}$

$\boxed{\text{dimensions } \sqrt{6}\text{ in} \times \dfrac{\sqrt{6}}{2}\text{ in}}$

(16)

max volume

$2x + 2y = 39 \rightarrow y = \dfrac{39 - 2x}{2}$

height $= y$

$x = 2\pi r \Rightarrow r = \dfrac{x}{2\pi}$

$V = \pi r^2 h = \pi\left(\dfrac{x}{2\pi}\right)^2 y$

$V = \pi\left(\dfrac{x}{2\pi}\right)^2\left(\dfrac{39 - 2x}{2}\right)$

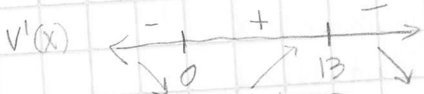
$V = \pi \cdot \dfrac{x^2}{4\pi^2}\,\dfrac{(39 - 2x)}{2}$

$V = \dfrac{1}{8\pi}(39x^2 - 2x^3)$

$V'(x) = \dfrac{1}{8\pi}(78x - 6x^2) = 0$

$-\dfrac{6x}{8\pi}(13 - x) = 0$

$x = 0 \quad x = 13$



max when $x = \boxed{13\text{ cm}}$

$y = \dfrac{39 - 2(13)}{2} = \boxed{\dfrac{13}{2}\text{ cm}}$

(17) $V = \dfrac{4}{3}\pi r^3 \qquad \dfrac{dV}{dt} = 5\,\dfrac{\text{cm}^3}{\text{sec}}$

find $\dfrac{dA}{dt}$ when $\dfrac{dV}{dt} = 256\pi/3\text{ cm}^3$

$$\frac{256\pi}{3} = \frac{4\pi r^3}{3}$$

$$256\pi = 4\pi r^3$$

$$\frac{256}{4} = r^3$$

$$64 = r^3$$

$$r = \sqrt[3]{64} = 4$$

$$A = 4\pi r^2$$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

Need $\frac{dr}{dt}$! Back to $\frac{dV}{dt}$:

$$\frac{dV}{dt} = \frac{4\pi}{3} \cdot 3r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$5 = 4\pi (4)^2 \frac{dr}{dt}$$

$$\frac{5}{64\pi} = \frac{dr}{dt}$$

$$\frac{dA}{dt} = 8\pi(4)\left(\frac{5}{64\pi}\right)$$

$$\frac{dA}{dt} = \boxed{\frac{5}{2} \frac{cm^2}{sec}}$$

(18) Volume $= x^2 y = 600\ cm^3$

$$\frac{dA}{dt} = 8 \frac{cm^2}{sec}$$

Find the rate at which the height of box is decreasing when sides are 18 cm $= x$

Find $\frac{dy}{dt} < 0$.

$$A = x^2$$

$$\frac{dA}{dt} = 2x \frac{dx}{dt} \Rightarrow 8 = 2(18)\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{8}{36} = \frac{2}{9}$$

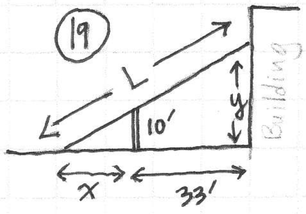
$$Y = \frac{600}{x^2} = 600x^{-2}$$

$$\frac{dy}{dt} = -1200x^{-1}\frac{dx}{dt} = \frac{-1200}{x}\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{-1200}{18} \cdot \frac{2}{9} = \frac{-2400}{162}$$

$$\frac{dy}{dt} = \boxed{\frac{-400}{27} \frac{cm}{sec}}$$

(19)



Find shortest L that will reach from ground to building. nearest tenth.

$$\begin{cases} (x+33)^2 + y^2 = L^2 & \text{Pythagorean: minimize } L \\ \frac{10}{x} = \frac{y}{x+33} & \text{Similar triangles} \end{cases}$$

$$\frac{10x + 330}{x} = \frac{xy}{x}$$

$$y = 10 + 330x^{-1}$$

$$(x+33)^2 + (10 + 330x^{-1})^2 = L^2$$

$$2L \cdot L'(x) = 2(x+33) + 2(10 + 330x^{-1})(-330x^{-2})$$

$$L \cdot L'(x) = x + 33 + \left(10 + \frac{330}{x}\right)\left(\frac{-330}{x^2}\right)$$

$$L \cdot L'(x) = x + 33 - \frac{3300}{x^2} - \frac{108900}{x^3}$$

$L \neq 0$ ever

numerator

$$L' = x + 33 - \frac{3300}{x^2} - \frac{108900}{x^3} = 0$$

$$x^4 + 33x^3 - 3300x - 108900 = 0$$

$$x^3(x+33) - 3300(x+33) = 0$$

$$(x+33)(x^3 - 3300) = 0$$

$$x = -33 \qquad x^3 = 3300$$

$$x = \sqrt[3]{3300}$$

L': (−) (+)

$\sqrt[3]{3300} \approx 14.8$

min

$$L(\sqrt[3]{3300}) = \sqrt{(\sqrt[3]{3300} + 33)^2 + \left(10 + \frac{330}{\sqrt[3]{3300}}\right)^2}$$

round to tenth, hallelujah.

$\boxed{L \approx 57.7\ ft}$

Math 250 Review and Practice

1) Consider the region bounded by $y = e^x, y = 2, x = 0$.

a. Find the area of this region. Evaluate this integral by hand, giving your answer in exact form.

b. Find the volume of the solid formed if this region is revolved about the $x-$axis. Evaluate this integral by hand, giving your answer in exact form.

c. SET UP AN INTEGRAL to find the volume of the solid formed if this region is revolved about the line $y=2$.

d. SET UP AN INTEGRAL to find the volume of the solid formed if this region is revolved about the $y-$axis.

e. SET UP AN INTEGRAL to find the volume of the solid formed if this region is revolved about the line $x = 4$.

f. Let the region described above be the BASE of a solid, whose cross-sections perpendicular to the x – axis are EQUILATERAL TRIANGLES. SET UP AN INTEGRAL TO find the volume of this solid.

2) (Calculator required) Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$.

Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about:

a. The line $y = -3$.

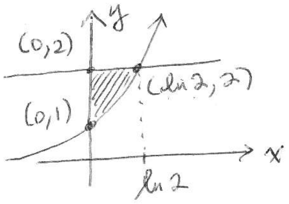
b. The y axis.

c. The line $x = 4$

Math 250 **Review and Practice** Volumes with logs and exp

1) Consider the region bounded by $y = e^x, y = 2, x = 0$.

a. Find the area of this region. Evaluate this integral by hand, giving your answer in exact form.



$e^x = 2$

$x = \ln 2$

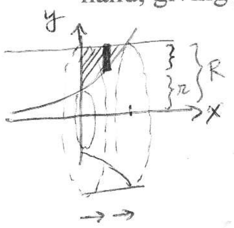
$$\text{Area} = \int_{x=0}^{\ln 2} (2 - e^x)\,dx$$

$$= (2x - e^x)\Big|_0^{\ln 2}$$

$$= (2\ln 2 - e^{\ln 2}) - (2\cdot 0 - e^0)$$

$$= 2\ln 2 - 2 - 0 + 1 = \boxed{2\ln 2 - 1}$$

b. Find the volume of the solid formed if this region is revolved about the $x-$axis. Evaluate this integral by hand, giving your answer in exact form.



washers $\pi R^2 - \pi r^2$

$$\int_{x=0}^{\ln 2} \pi(2)^2 - \pi(e^x)^2\,dx$$

$$= \pi\left[\int_{x=0}^{\ln 2} 4\,dx - \frac{1}{2}\int_0^{\ln 2} 2e^{2x}\,dx\right]$$

$u = 2x$

$du = 2dx$

$u_1 = 2(0) = 0$

$u_2 = 2\ln 2 = \ln 2^2 = \ln 4$

$$= \pi\left[4x\Big|_0^{\ln 2} - \frac{1}{2}\int_0^{\ln 4} e^u\,du\right]$$

$$= \pi\left[(4\ln 2 - 4(0)) - \left(\frac{1}{2}e^u\Big|_0^{\ln 4}\right)\right]$$

$$= \pi\left[4\ln 2 - \frac{1}{2}\left(e^{\ln 4} - e^0\right)\right]$$

$$= \pi\left[4\ln 2 - \frac{1}{2}(4 + 1)\right]$$

$$= \boxed{\pi\left(4\ln 2 - \frac{3}{2}\right)}$$

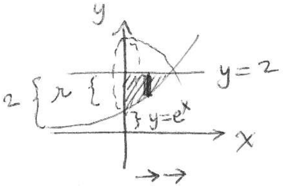
check by GC: ≈ 3.9979554 ✓

shells

$\int_{y=1}^{2} 2\pi y \cdot \ln y\,dy$

can't integrate

c. SET UP AN INTEGRAL to find the volume of the solid formed if this region is revolved about the line $y = 2$.



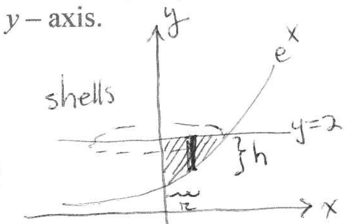
disks πr^2

$$\int_{x=0}^{\ln 2} \pi(2 - e^x)^2\,dx = \boxed{\pi\int_0^{\ln 2} (2 - e^x)^2\,dx}$$

shells $2\pi r h$

$$= \boxed{2\pi\int_{y=1}^{2} y(\ln y)\,dy}$$

d. SET UP AN INTEGRAL to find the volume of the solid formed if this region is revolved about the $y-$axis.

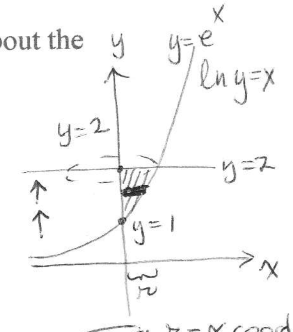


Shells $2\pi r h$

$$\int_{x=0}^{\ln 2} 2\pi x(2 - e^x)\,dx = \boxed{2\pi\int_0^{\ln 2} x(2 - e^x)\,dx}$$

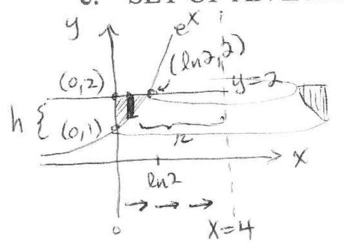
disks πr^2

$$\int_{y=1}^{2} \pi(\ln y)^2\,dy = \boxed{\pi\int_1^2 (\ln y)^2\,dy}$$



r = x coord in terms of y

e. SET UP AN INTEGRAL to find the volume of the solid formed if this region is revolved about the line $x = 4$.



shells $2\pi r h$

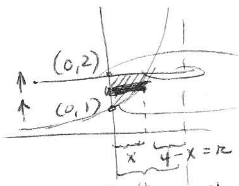
$$= \int_{x=0}^{\ln 2} 2\pi (4-x)(2-e^x)\,dx$$

$$= \boxed{2\pi \int_{0}^{\ln 2} (4-x)(2-e^x)\,dx}$$

washers $\pi R^2 - \pi r^2$

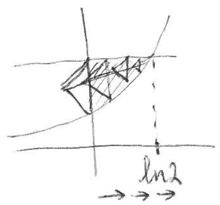
$$\int_{y=1}^{2} \pi (4)^2 - \pi(4-\ln y)^2\,dy$$

$$= \boxed{\pi \int_{1}^{2} 16 - (4-\ln y)^2\,dy}$$



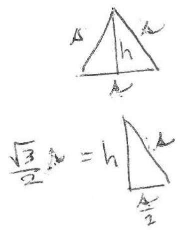
$x = \ln y$

f. Let the region described above be the BASE of a solid, whose cross-sections perpendicular to the x – axis are EQUILATERAL TRIANGLES. SET UP AN INTEGRAL TO find the volume of this solid.



$$\int_{x=0}^{\ln 2} A(x)\,dx$$

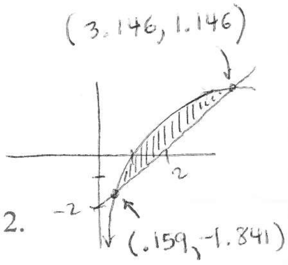
$$= \boxed{\int_{x=0}^{\ln 2} \frac{\sqrt{3}}{4}(2-e^x)^2\,dx}$$



$A = 2 \cdot \frac{1}{2} \cdot \frac{s}{2} \cdot \frac{\sqrt{3}}{2} s$

$A(s) = \frac{\sqrt{3}}{4} s^2$

Area of equilateral Δ

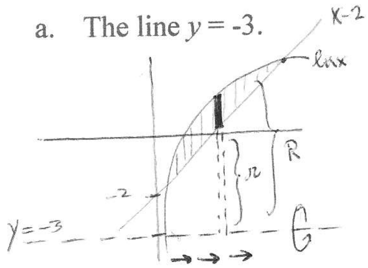


2) (Calculator required) Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$.

on GC: $\ln x = x - 2$

Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about:

a. The line $y = -3$.



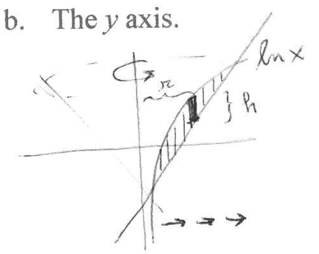
washers: $\pi R^2 - \pi r^2$

$$\int_{x=.159}^{3.146} \pi(\ln x - (-3))^2 - \pi(x-2-(-3))^2\,dx$$

$$= \boxed{\pi \int_{.159}^{3.146} (\ln x + 3)^2 - (x+1)^2\,dx}$$

shells
see next page

b. The y axis.

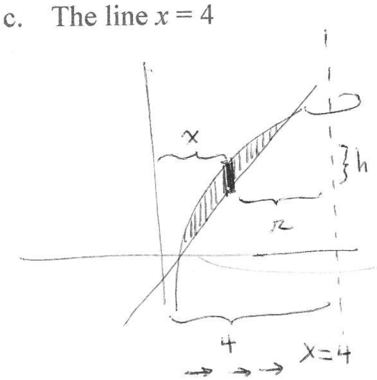


shells: $2\pi r h$

$$\int_{.159}^{3.146} 2\pi x(\ln x - (x-2))\,dx = \boxed{2\pi \int_{.159}^{3.146} x(\ln x - x + 2)\,dx}$$

washers
see next page

c. The line $x = 4$



shells: $2\pi r h$

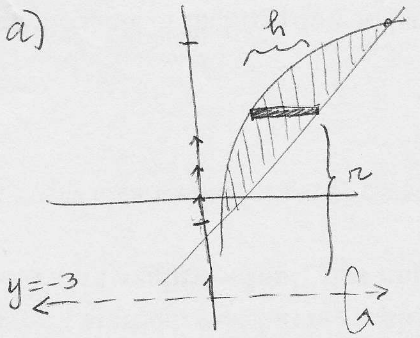
$$\int_{x=.159}^{3.146} 2\pi(4-x)(\ln x - (x-2))\,dx$$

$$= \boxed{2\pi \int_{.159}^{3.146} (4-x)(\ln x - x + 2)\,dx}$$

washers
see next page

Math 250 Integrating in y.

(2) a)



$\ln x = y \qquad y = x-2$

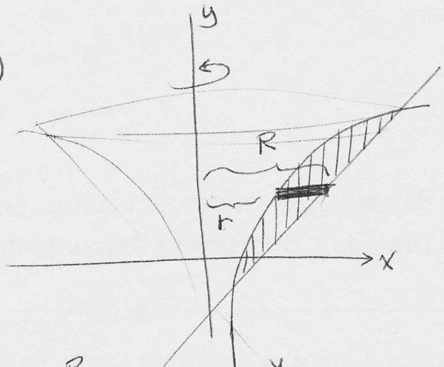
$x = e^y \qquad y+2 = x$

shells: $2\pi r h$

$$= \int_{y=-1.841}^{1.146} 2\pi(y - -3)(y+2-e^y)\,dy$$

$$= \boxed{2\pi \int_{-1.841}^{1.146} (y+3)(y+2-e^y)\,dy}$$

b)



R $\quad$ $y+2=x$

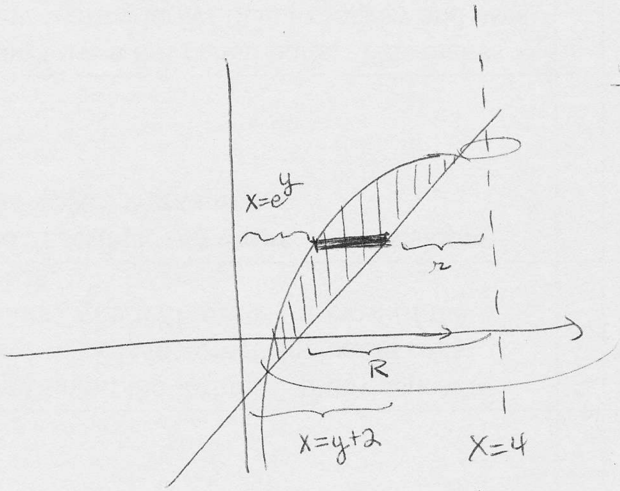
$x=e^y$ $\quad$ r

washers $\pi R^2 - \pi r^2$

$$\int_{y=-1.841}^{1.146} \pi(y+2)^2 - \pi(e^y)^2\,dy$$

$$= \boxed{\pi \int_{-1.841}^{1.146} (y+2)^2 - e^{2y}\,dy}$$

c)



washers: $\pi R^2 - \pi r^2$

$$\int_{y=-1.841}^{1.146} \pi(4-e^y)^2 - \pi\left(4-(y+2)\right)^2\,dy$$

$$= \boxed{\pi \int_{-1.841}^{1.146} (4-e^y)^2 - (2-y)^2\,dy}$$